Efficient Algorithm for Coupled Problem Simulation of High Dynamic Electromagnetic Actuators

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Methodology for efficient coupled problem simulation of the linear electromagnetic actuator with high dynamic response is proposed and experimentally validated in the framework of multi-coil permanent magnet actuator in the valve operation. Major goal of presented research and development is to balance computational effort of the algorithm with complexity of computational problem. Proposed algorithm then enables efficient solution of coupled problem for model-based controller and state observer of the system.

Index Terms—Actuators, numerical simulation, finite element analysis, interpolation, state estimation, control systems

I. INTRODUCTION AND MOTIVATION

MATHEMATICAL MODELING of highly dynamic electromechanical problems represents very demanding computational task, especially in the case when solution of the model has to be performed by controller or observer with relatively low computational power.

Nevertheless, state-of-the-art applications of linear electromagnetic actuators in the complex cyber-physical systems call for utilization of advance controllers and state observers which require very efficient solution of multiphysics model [1], [2]. For example, fully controlled movement of the electromagnetic actuator plunger or fault detection and isolation of its operation addresses cutting-edge issues in the field and represents also major motivation for presented research and development.

II. DESCRIPTION OF TECHNICAL PROBLEM

The proposed methodology was designed originally for a unique bistable electromagnetic actuator in valve operation, but it is usable for an arbitrary linear electromagnetic actuator, especially in multi-coil design of the actuator with permanent magnets. Figure 1 shows an arrangement of the developed laboratory prototype and explains its basic operation.

Fig. 1. Simplified arrangement of discussed electromagnetic actuator in valve operation. Left part shows fully closed and right part depicts fully opened valve, but opening is initiated by excited coil. Stable positions are secured by permanent magnets (orange loops illustrate magnetic flux generated by PM), opening and closing of the valve actuator is initiated by excitation of the appropriate field coil by DC current (red loops illustrate magnetic flux generated by coils).

Analysis of the magnetic field inside the actuator represents critical issue for simulation of the valve operation. Fast calculation based on equivalent magnetic circuit is traditional computational approach used for design [3] and also by the

control algorithms, however, this approach generally strongly reduces complexity of the model. On the other hand, solution of the nonlinear mathematical model of the magnetic field (1) is demanding and also consuming much time.

Another common solution removes a huge part of the calculation effort of the control algorithm and uses precalculation of required quantities [2]. Characteristics of the force $F(\delta, v, i_1, \ldots, i_n)$ acting on the plunger (δ represents current position of the plunger, v is plunger velocity and i_j stands for field coil currents), inductance $L(\delta, v, i_1, \dots, i_n)$ of each field coil and also the induced voltage $u_i(\delta, v, i_1, \ldots, i_n)$ are stored in lookup tables. Nevertheless, complex representation of the model (1) collides with the demand for amount of sampling.

The proposed methodology takes advantage of precalculation of the magnetic flux $\Phi(\delta, v, i_1, \dots, i_n)$ on irregular grid of a multidimensional parameter space (each demanded parameter of the model represents just one dimension: $\delta, v, i_1, \ldots, i_n$). Afterwards, the parameter space is interpolated by multivariate algorithm and all required quantities are then calculated directly from the characteristic of the magnetic flux.

III. FORMULATION OF MATHEMATICAL MODEL

The distribution of magnetic field inside actuator for any position of the movable plunger is generally described by the partial differential equation in the form

curl
$$
(\mu(|\mathbf{B}|)^{-1} (\text{curl } \mathbf{A} - \mathbf{B}_{r})) - \gamma \mathbf{v} \times \text{curl } \mathbf{A} +
$$

 $+ \gamma \frac{\partial \mathbf{A}}{\partial t} = \mathbf{J}_{\text{ext}} ,$ (1)

where μ denotes the nonlinear magnetic permeability, \boldsymbol{A} stands for the magnetic vector potential, \bf{B} and \bf{B}_r represent the magnetic flux density and remanent magnetization, respectively, γ is the electric conductivity, v denotes the velocity of plunger and J_{ext} represents the density of the field current in the appropriate field coil.

The transient response of current i in any of the field coils is given by an ordinary differential equation in the form

$$
U_0 = Ri + \frac{\mathrm{d}\Phi(\delta, i)}{\mathrm{d}t},\tag{2}
$$

where U_0 represents the source voltage, R is its resistance and $\Phi(\delta, i)$ is the flux linkage.

The total derivative of flux linkage $\Phi(\delta, i)$ with respect to t can be simplified by indirect dependencies by equation

$$
\frac{\mathrm{d}\Phi(\delta,i)}{\mathrm{d}t} = \frac{\partial\Phi(\delta,i)}{\partial\delta}\frac{\partial\delta}{\partial t} + \frac{\partial\Phi(\delta,i)}{\partial i}\frac{\partial i}{\partial t},\tag{3}
$$

where the first component on the right-hand side of the equation represents voltage induced by plunger movement and the derivative of the flux linkage Φ with respect to current i represents the inductance of appropriate field coil.

The dynamics of the movable plunger is described by the motion equations in the form

$$
m\frac{\mathrm{d}v}{\mathrm{d}t} = F_{\mathrm{m}} + \sum_{i} F_{i} , \quad v = \frac{\mathrm{d}\delta}{\mathrm{d}t} , \tag{4}
$$

where m denotes the mass of the plunger, F_m is force produced by magnetic field and $\sum_i F_i$ stands for superposition of additional forces depending on the valve operation conditions (friction forces, gravity force, etc.). Magnetic force F_m is than calculated by formula

$$
F_{\rm m} = \int_0^i \frac{\partial \Phi(\delta, i)}{\partial \delta} \, \mathrm{d}i \,. \tag{5}
$$

From mechanical viewpoint, rebounds of the plunger from elastic intents are of huge importance. In general case, the coefficient e of restitution can be used and the impact is than described in the way $v_{\rm b} = e \cdot v_{\rm a}$, where $v_{\rm b}$ and $v_{\rm a}$ represents velocities of the plunger before and after impact.

IV. METHODOLOGY VALIDATION

Validation and verification ot the methodology was performed by simulation experiments and laboratory measurements on the discussed electromagnetic actuator prototype (see Fig. 1). The nonlinear magnetic model (1) was solved as a 2D axi-symmetric problem and numerical solution was realized by a higher-order finite element method whose algorithms are implemented in our own code Agros2D.

Magnetic flux $\Phi(\delta, v, i_{o}, i_{c})$ (*i_o* and *i_c* stand for opening and closing currents) was calculated on predefined 4D irregular grid interpolated by multivariate nearest-neighbour algorithm because of the allocated memory (total allocated memory of the array is 44.6 KiB in comparison of 13.1 KiB for computation algorithm). Transient response of the field coil currents i_0 , i_c (2) and plunger dynamics defined by δ and v (4) is then calculated by *the* Runge-Kutta method in the control algorithm.

Fig. 2. Plunger dynamics simulation performed by proposed algorithm for actuator prototype. Total closing time is equal to $t_c = 35$ ms and plunger movement time is $t_m = 17$ ms (results are comparable with Fig. 3).

Experimental validation of the proposed algorithm was performed by the optical measurement of plunger dynamics by high-speed camera. Figure 2 shows simulation of actuator opening performed by proposed algorithm and for comparison Figure 3 shows results of the measurement. Total opening time is $t_0 \approx 36 \,\text{ms}$ (calculated $t_0 = 35 \,\text{ms}$) and plunger movement time is $t_m \approx 19 \,\text{ms}$ (calculated $t_m = 17 \,\text{ms}$). Measured plunger rebound is approximately 0.5 mm in comparison with 0.6 mm obtained by calculation.

Fig. 3. Dynamics of movable plunger captured for opening mode of valve prototype. Measured plunger movement time is $t_m \approx 19$ ms and total opening time $t_0 \approx 36$ ms with consideration of plunger rebound. Captured frames were processed by Sobel operator for post-processing edge detection.

Major utilization of the proposed algorithm – fast simulation of the actuator high dynamic cycle operation in the modelbased controller is demonstrated in Fig. 4.

Fig. 4. Simulation of cycle operation of the actuator prototype performed by proposed algorithm. Simulation demonstrates simple elimination of the plunger rebound based on partial excitation of fields coil throughout whole operation.

V. CONCLUSION

Efficient methodology for coupled problem simulation of high dynamic linear electromagnetic actuator was proposed and validated on the laboratory prototype. Proposed algorithm enables to reduce computational effort while maintaining complexity of the model.

Computation of the flux linkage, rules for sampling of parameter space on irregular grid and also algorithmic uncertainty (uncertainty comes from approximation errors) will be discussed in-depth in the full paper.

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